Stochastic Frequency Distribution Analysis as Applied to Mottle Measurement

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Abstract

In any mottled digital image, the Stochastic Frequency Distribution Analysis (SFDA) algorithm measures the two dimensional rate of change in luminance values, or transitions between light and dark, from one area to the next. It is also responsive to the relative number of these transitions.

In print applications SFDA mathematically describes the uniformity of ink transfer to the substrate surface by producing a number proportional to the visible, or sub-visible, degree of mottle. It also provides horizontal and vertical mottle orientation measurements.

SFDA is used measure a wide variety of mottled images including, ink jet print, toner adhesion, toner quality, paper formation, ink penetration, offset and rotogravure.

Introduction

Mottle is pervasive. Everything the human eye sees is mottle. When the mottle becomes organized to form objects or patterns human cognition begins to seek or assign a name to the objects formed. What is left after the objects are identified is mottle.

Texture is the primary and smallest visible component of mottle. Without texture a visible surface would be smooth and featureless. Thus at some level of magnification and illumination, with the possible exception of a perfect mirror, all visible surfaces have texture.

The human image processor operates almost instantaneously in a three dimensional environment composed of the luminance intensities contained in the length and height of the viewing area. SFDA attempts to emulate the human intellect by analyzing the content of a digital image. It first determines the properties of the image texture and then the spatial distribution of this texture. The digital image resolution, the number of pixels per unit measure, is an integral part of the SFDA measurement, and, at high resolutions, can allow the mottle measurement to operate in the sub-visible range.

Stochastic Frequency Distribution Analysis

The word stochastic derives from the Greek word *stochas*, a post. As the ancient archers used a post for practice, it also

means target. From a digital image, SFDA extracts square contiguous target areas of exactly the same pixel dimensions. The number and size of the targets is determined by the specific application.
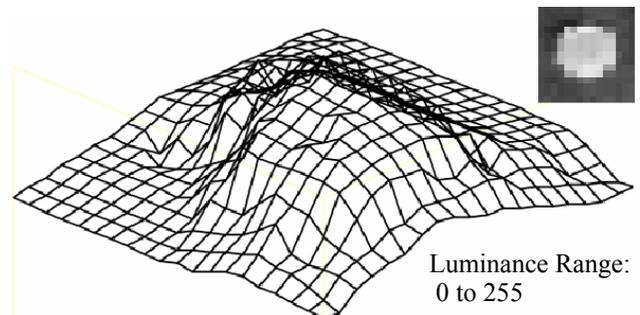

Figure 1. 3D luminance map of gray dot shown in upper right.

Figure 1 schematically illustrates a three-dimensional graph of the luminance values present in an image of a light dot on a gray background shown in the upper right. The dot was chosen for this example because its pixel luminance values vary across both the width and length of the inspection area. At some level of magnification this illustration could be representative of an ink jet print dot, a half-tone dot, or a button on the jacket in the picture of a small boy.

Figure 2 illustrates the basic measurement in SFDA: A small two-dimensional sample, in this case, a 5 pixel x 5 pixel target, is isolated from the image. The two-dimensional standard deviation, "s", is computed for the
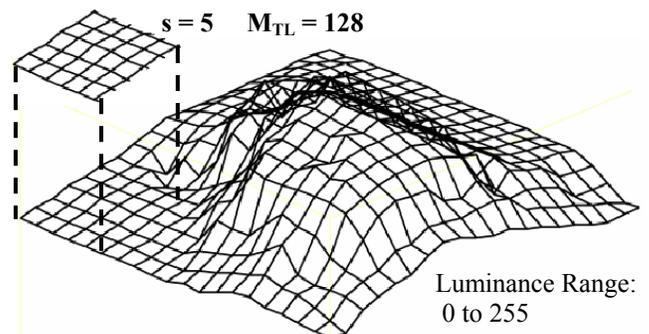

Figure 2. The standard deviation (s) of the luminance values in a 5 pixel x 5 pixel target area of the dot background.

pixel luminance values in the square target as:

$$s = \sqrt{\frac{\sum (P_L - M_{TL})^2}{n}} \qquad (1)$$

Where P_L is the pixel luminance, M_{TL} is the mean luminance for the pixels in the target, and n is the number of pixels in the target.

As shown in Figure 3, moving the target area to a contiguous sampling position at the edge of the dot, changes the standard deviation of the pixel luminance values from 5 to 21. Because the target is now positioned over an area of transition, there is a wider variety of pixel luminance values within it. The calculation of "s" is a highly sensitive

s = 21 M_{TL} = 145
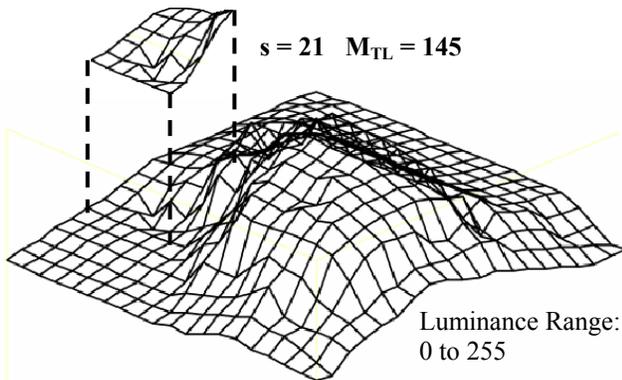
Luminance Range:
0 to 255

Figure 3. The standard deviation (s) of the luminance values in a 5 x 5 pixel target of the transition from the background.

indicator of the two-dimensional relative rate of change, or transition, between the light and dark areas and is the basis of the SFDA mottle measurement.

When positioned over an area of sharp transition as shown in Fig. 4A, "s" will rise indicating this sharp transition and will be lower with the more gradual transition as in Fig. 4B. The measurement could be confused when the pattern of the same pixel luminance values is randomized as in Fig. 4C. In this case the measurement
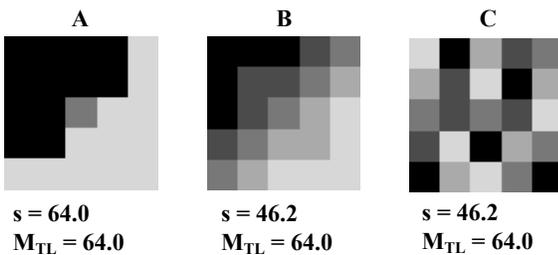

A	B	C
s = 64.0	s = 46.2	s = 46.2
M_{TL} = 64.0	M_{TL} = 64.0	M_{TL} = 64.0

Figure 4: Sharp and gradual transitions of a 5 x 5 pixel target showing same mean luminance value but different standard deviation (s). "C" shows "B" randomized.

could indicate the area is part of a gradual transition or, if it dominates the image, be part of the overall background. It is possible for subtle changes in Fig. 4C to indicate the presence of a mottle pattern such as found in many paper formation tests.

Image Resolution

Fig. 4 also serves to illustrate the need to have sufficient resolution in the digital image detail to quantify the transitions in luminance values. Experiments have shown that the image texture can be sub-visible but yet its effects are observed unassisted. In these cases, a higher than normal resolution was required to provide sufficient detail to identify transitions between light and dark areas using a small 5 x 5 target size.

The optimal settings for image resolution and target size are determined by examining the specimen at various resolutions, determining the texture that is causing the perceived mottle, and then setting the target dimensions necessary to indicate the transitions. For example, Fig. 3 illustrates a 5 x5 target will indicate the change from dark to light and Fig. 2 shows this same target size works in the background valley and the relatively smoother area in the center of the dot. Zooming a digital image will visibly indicate when these transitions are apparent, the required resolution, and the target dimensions necessary to measure them.

Texture Target Size

The physical dimensions of the texture target are

5 pixels | 5 pixels | 5 pixels | 5 pixels | 5 pixels

255

128

0

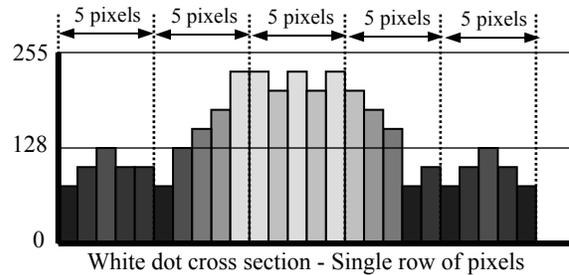White dot cross section - Single row of pixels

Figure 5: Sub-division of an image profile into sections that demonstrate the difference in luminance value rates of change.

determined by the image resolution and the rate at which the underlying pixel luminance values change from one area to the next.

For example, Fig. 5 illustrates how the pixel dimensions of the texture target are set to be sensitive to the variations present in an image. The profile shown may be taken from that of a dot or a white line on a gray background. The pixel limits are set to measure the transition and plateau areas. In practice the SFDA algorithm will convert the linear pixel count into the square shown in Figs. 2 & 3. The 5 x 5 target is used in these illustrations as a matter of convenience and consistency. In practice the square target can be any dimension that will differentiate between the plateau and transition areas using the standard deviation of the luminance values contained.

When used in a test environment, the target dimensions and the resolution used to acquire the image, must be determined for specific test conditions and fixed for the particular application as shown in Fig. 13.

Diversity of image intensities

The mean (M_{TL}) luminance value in all three examples in Fig. 4 remains the same thus leading to the conclusion that the mean a less reliable indicator of the transitions present in an image than the standard deviation, "s". The target mean, however, is an indicator of the diversity of visual intensities or local area contrasts. While the "s" will indicate the presence of a change in intensities the "M_{TL}" describes the overall visual impact of the area inspected; "M_{TL}" provides information about the relative visual impact of the light and dark areas present.
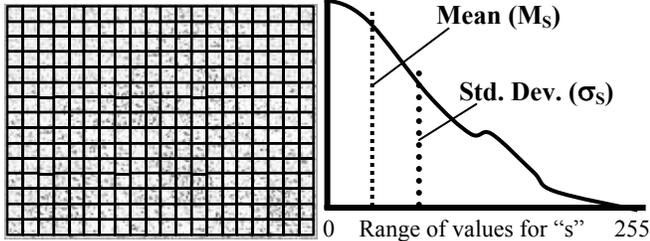

Figure 6: Mottled image with contiguous targets overall.   *Figure 7: "s" values for Fig. 6. Overall sample area: Texture.*

Target Sampling of the Image, Database Creation

As shown in Figs. 6 & 7, SFDA first samples the entire digital image in a regular pattern of contiguous target areas. As the target areas are measured data are stored in two databases:

1- Standard deviation, "s"
2 - Mean, "M_{TI}"

To enable subsequent examination of spatial distribution requiring extraction of specific areas of interest within the image, these databases are dimensioned the same as the overall target acquisition pattern.
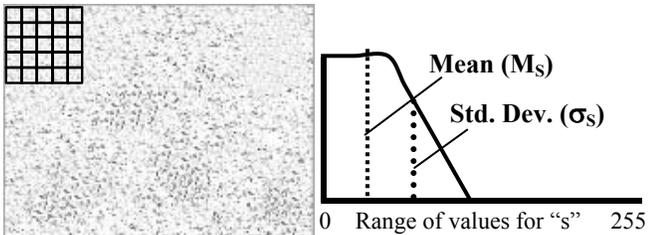

Figure 8: A group of 25 targets sampling an area of low mottle.   *Figure 9: "s" values for Fig. 6 image sample area: low mottle.*
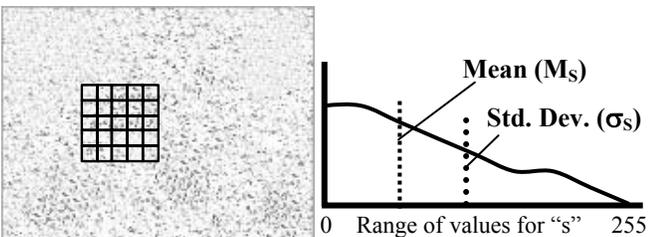

Figure 10: A group of 25 targets sampling an area of high mottle.   *Figure 11: "s" values for Fig. 8 image sample area: high mottle.*

Mottle Number

Given the standard deviation, "s", of the pixel intensity within the target area is an indicator of the two dimensional rate of change in pixel luminance for that target. It follows then that the degree of variation among the "s" values is an indicator of the degree of uniformity among the targets. It follows likewise for the mean, "M_{TI}", that its variance is an indicator of the degree of uniformity in the image luminance. Three calculations are made using these target data:

σ_S = The standard deviation of the "s" values
M_S = The mean of the "s" values
σ_m = The standard deviation of "M_{TL}" values

These calculations are then combined to compute the mottle number for the area of the image inspected as:

$$Mottle = K \ x \ (\sigma_S \ x \ M_S \ x \ \sigma_m) \qquad (2)$$

where K is a scaling factor.

SFDA Mottle – Spatial Distribution

Using their bounding dimensions, selected areas of the image, or the entire image, can be analyzed by extracting the area base information from the databases.

Figure 8 shows a mottled image in which the texture is unevenly distributed. In the upper left corner of the image, where the mottle is low, 25 targets have been overlaid in a square pattern as an area of interest. The "s" values for the bounded area were extracted from the database and schematically plotted in the histogram shown in Figure 8 with their mean, M_S, and standard deviation,σ_S .

Figure 10 shows a comparison to an area of relatively high mottle in the same image. Figure 11 shows the associated histogram of the "s" values within the bounding area. Note the differences in the frequency distribution and the relative positions of the mean, M_S, and the standard deviation, σ_S. These generalized trends persist through all actual examinations of mottle; As the image becomes less mottled the σ_S & M_S of the "s" values decrease. This is not true for the σ_m which responds primarily to the area covered by the luminance values present and can yield the same number under a variety of mottle conditions.

Figure 12: Spatial distribution analysis showing a typical second target size composed of 25 smaller texture targets.

Spatial Distribution of Texture Mottle

Figs. 9 & 11 illustrate the fact that the texture of an image can vary from area to area. To measure the degree to which this variation occurs, the overall image is uniformly

subdivided into larger areas as shown in Fig. 12. Each of these larger areas contains the same number of the small texture targets and each larger area is analyzed individually. The Mottle Number for the smaller targets contained in the larger areas is calculated using Eq.2. The resulting Mottle Number for each larger area is then used to calculate the following statistics:

σ_O = Standard Deviation of Large Target Mottle numbers
M_O = Mean of Large Target Mottle numbers

These calculations are then combined to compute the mottle number for the spatial distribution determined by the larger target size:

$$Spatial\ Mottle = K\ x\ (\sigma_O\ x\ M_O) \qquad (3)$$

where K is a scaling factor.

Texture - Physical Dimensions & Image Resolution

Fig. 13 lists some of the settings for tests that have been run using SFDA. The physical measurement of the texture in each application dictates both the image resolution and the size of the texture measurement target.

SFDA Test	Resolution ppi	Pict. Pt Spacing	Texture Target mm sq	pp sq
Formation	200	0.127 mm	1.02	8
IGT Strips	200	0.127 mm	0.64	5
Ink Jet Mottle	500	0.051 mm	0.51	10
Prufbau Strips	500	0.051 mm	0.25	5
RIT Back Trap & Wet Trap	500	0.051 mm	0.25	5
Offset Print 25 mm squares	500	0.051 mm	0.25	5

Figure 13: A table of SFDA settings and picture point spacing.

Ink Jet Solid Print Uniformity

A major consideration in the evaluation of paper designed for ink jet print is the ability of the paper to produce images that are appealing to the eye. Properties such as wicking and color to color bleed are routine quality control measurements made in all paper mill quality control laboratories producing these papers. Many of these labs use instrumentation designed to measure the quality produced from standardized images[1] generated on ink jet printers. These patterns are designed to stress the ink to paper interaction and to facilitate measurement with automated image analysis programs.

The measurements reported by this instrumentation are heavily dependent upon the performance of the ink jet cartridges used in the printer. In order to measure the paper properties it is necessary to use cartridges that perform uniformly. The mottle algorithm discussed in the preceding will measure the ability of the cartridge to perform to a standard.

A magnified section of a larger print is shown as an example of good and poor cartridge performance in Fig 14. Section "B" shows the streaking or banding produced by

poor jet performance. Fig 14 also shows one of the small targets at work on the specimens. This target is only one of thousands that will be applied to the full images of these solid print areas (See Figs. 5 & 6). The data derived from these targets is then reduced to a single mottle number using equation (2).
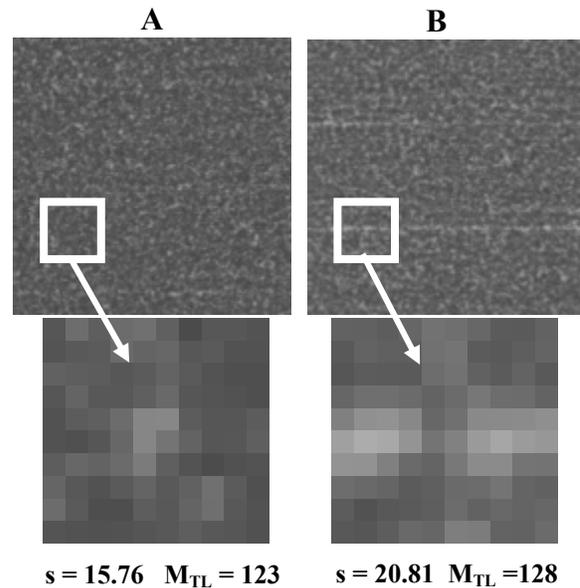

$s = 15.76 \quad M_{TL} = 123$ $s = 20.81 \quad M_{TL} = 128$

Figure 14: Areas printed as solids using a cyan ink jet cartridge. "A" appears to be fairly uniform;" B" clearly shows evidence of banding or streaking.

Horizontal and Vertical Spatial Orientation

The SFDA algorithm creates texture databases mapped the same as the image pixel array, row and column variability information can be extracted as a secondary source of mottle data. This is particularly useful in measuring mottle in ink jet print as it will tend to pinpoint poor performing jets.

References

1. Roy Rosenberger, A Quantitative Method of Relating Polychromatic Print Quality to the Human Evaluator, *Proc. TAPPI / PAPTAC 2000 Printing & Graphic Arts,* pg 37. 2000

Biography

Roy Rosenberger received his B.A. from Pennsylvania State University in State College in 1958 and a M.B.A. in Finance from Iona College in New Rochelle, NY in 1980. He spent most of his career serving the pulp and paper industry and has been granted several patents ranging from process control to fluid dynamics. Most recently he has

concentrated upon the development of image analysis measurement software for paper and print quality control.