

**Topography, Visible Print Mottle, Half-Tone Mottle and Paper Formation,  
Pattern Measurement with the  
Verity IA Stochastic Frequency Distribution Analysis (SFDA) Algorithm  
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The Stochastic Frequency Distribution Analysis (SFDA) is usually employed in the measurement of surface patterns such as, topography, visible solid tone print mottle, half-tone print mottle and paper formation. SFDA is a digital algorithm that operates only on digital images that can be acquired by any means. The original image may be poly-chromatic but must be processed to produce a mono-chromatic image for analysis. Its pixel (picture elements) luminance values (PLV) must vary from zero (0) for black to any number equal to or greater than 255 for white. The image content must be intended to be spatially uniform as the algorithm will measure the degree of spatial dispersion within the image PLV on a scale where zero (0) represents a perfectly smooth or uniformly dispersed subject matter lacking any features or texture in which case the PLV all have the same value.

SFDA employs square targets (Often referred to as tiles) that can be a range of different sizes, to measure the uniformity of a random pattern's PLV spatial distribution. When measuring visible phenomenon such as print mottle, the targets, if they were actual, would usually be visible at normal viewing distance. When measuring sub-visible features such as topography and half-tone mottle, the SFDA targets would not be visible under normal viewing conditions.

### **Image Resolution**

The resolution of the original image to be analyzed must be high enough to record the features of the mottle or pattern to be measured. With visible print mottle this is the texture or minute disturbances reproduced only with resolutions equal to or greater than 236 pixels per centimeter (ppc), or 600 pixels per inch (ppi). Because the eye detects but averages together sub-visible feature luminance, higher resolution than normally expected is required to resolve print mottle.

Paper formation does not have visible texture as such, but does have wire marks that can obscure the formation measurement. In this case low resolution will minimize the impact of wire marks. The 60 ppc (150 ppi) is the recommended resolution for formation measurement. When measuring the sub-visible features in optical surface topography and the dots that make up half-tone print mottle, SFDA measurements require image resolutions of at least 472 ppc (1200 ppi).

### **Targets: Movement & Size**

The SFDA measurement employs square targets. These square targets are moved through the entire image following a regular traversing pattern. Typically starting at the upper right corner of the image, the target is moved one half its width to the right, stops, makes a measurement and then moves another half width, measures and moves, repeating the move-measure pattern until the edge of the image is reached. Another line of measurements is begun that is one half its height below the first or preceding line. This movement pattern continues until the bottom of the image is reached.

Like the resolution, the size of the target depends upon the pattern to be analyzed; usually multiple target sizes are employed depending upon the surface being analyzed. When a series of sizes is employed the first target size used determines the progression of target sizes that will be used. The target physical dimensions follow a binary progression, i.e. 2, 4, 8,...1024 (maximum), as multiples on the first target. When only one target is used there is no size progression.

The target physical size progression used for print mottle measurement in an image with a resolution of 236 ppc (600 ppi) includes the target widths: 0.677 mm, 1.355 mm and 2.709 mm. Similarly, formation measurements use a resolution of 59 ppc (150 ppi) and target widths of 1.355 mm and 2.709 mm. In these two examples the targets of 0.677 mm and 1.355 mm are visible; the observed pattern is visible.

Topography is sub-visible; its features cannot be seen with the naked eye. When measuring topography a resolution of 472 ppc (1200 ppi) is required and the range of targets used in the evaluation starts with 0.338 mm which is below the visible limit.

**Internal Rate of Change (IROC) within the primary target area:**

Up to this point the SFDA measurement resembles others that measure the variation spatial distribution but there is a difference, SFDA measures the Internal Rate of Change (IROC) within the image in addition to the variation in spatial distribution.

In the analysis, the target used is always a square, dimensioned 2 elements x 2 elements as shown in the flow chart below. As described above, the target dimensions are determined by the measurement to be made, e.g. visible print mottle, topography, roughness, formation, etc. At the initiation of a SFDA measurement, the physical size (length and width) of this primary target square and the four (4) equal contiguous squares elements within it (See flow chart below) are rationalized to the resolution of the image to be analyzed. The primary target length and width measurements are always in integral pixels dimensions.

After the primary square physical dimensions are rationalized, the individual Pixel Luminance Values of the Pixels (PLV) within each smaller square are averaged. The average is recorded in a two dimensional data base at a location corresponding to the physical location of the smaller square. The new data base elements will be the equivalent of a new image containing the average of the PLVs in the smaller squares within the primary target.

The question might be asked: “Would using a lower resolution to acquire the original image suffice instead of this averaging technique?” Empiric testing has shown that the human eye discerns the small variations that are sub-visible and, although the image may appear correct, the digital camera, when asked to operate a lower resolution, does not reproduce important details necessary to do a good analysis.

After the creation of this new image data base containing the PLV averages within the smaller square elements, the primary target is now moved one data base element horizontally. When the traverse is complete, the target is indexed down one target element and traversed one element at a time through the entire data base. This is the equivalent of moving through the original image one half (½) its physical width horizontally and one half (½) its physical height vertically through out the entire image.

### Calculations: SFDA

At each stop in the element by element traverse of the data base, the SFDA algorithm calculates:

1. The Internal Rate of Change (IROC) as the cross absolute differences in average luminance within the small squares in the primary target as shown in step five (5) in Figure 1. With this individual IROC, the algorithm updates the variables necessary to calculate:

The Standard Deviation ( $\sigma_{IROC}$ )

The Mean ( $M_{IROC}$ )

for the primary target IROCs at the completion of the data base traverse

2. The mean luminance value ( $Mean_{Ave 1-4}$ ) of the small squares average luminance values. With this individual mean value, the algorithm updates the variables necessary to calculate:

The Standard Deviation ( $\sigma_{Mean}$ )

### At the completion of the data base traverse

The SFDA Number is calculated:

$$SFDA\ Number = Constant \times \sigma_{IROC} \times M_{IROC} \times \sigma_{Mean}$$

### The measurement with a range of target sizes:

The results of the SFDA measurement in each target size within the preset range are averaged together for the final SFDA number. An image with a SFDA Number of “0” is the ideal, which occurs when no pattern or texture is exists in the image.

The SFDA measurement results from each size target are simply averaged:

$$Final\ SFDA\ Number = Constant \left( \sum_{1\ to\ N} SFDA / N \right)$$

Where N is the number of Target Sizes used in the analysis.